Solution

American University of Beirut
Mathematics Department
Math 204 Spring 2013-2014
Quiz II

Time: 70 min.

Name : _____ **ID#:** _____

Circle your problem solving section number below:

- **Instructor : Ms. Michella Bou Eid**

  Sec 1 : Th @ 3 :30 **Sec 2 : Th @ 2 :00**

- **Instructor : Ms. Joumana Tannous**

  Sec 4 : F @ 9 :00 **Sec 5 : F @ 10 :00** **Sec 6 : F @ 11 :00** **Sec 7: F @ 1 :00**

- **Instructor : Mrs Maha Itani-Hatab**

  Sec 8: M @ 1 :00 **Sec 9 : M @ 8 :00** **Sec 10: M @ 10 :00** **Sec 11: M @ 12 :00**

- **Instructor : Ms.Rana Nassif**

  Sec 12: W @ 1 :00 **Sec 13 : W @ 12 :00**

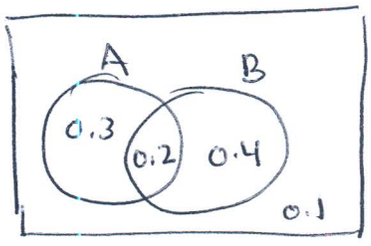
- **Instructor : Ms. Najwa Fuleihan**

  Sec 14 : T @ 8 :00 **Sec 15 : T @ 11 :00** **Sec 16 : T @ 9 :30**

1.	2.	3.	4.	5.	6.	7.	Final Grade

Answer each of the following questions. **(Justify your answer and show your work).**

1. If $P(A \cap B)' = 0.8$, $P(A' \cap B) = 0.4$ and $P(A) = 0.5$ then: (12 %)



a) $P(A \cap B) = 1 - P(A \cap B)' = 1 - 0.8$
$= \boxed{0.2}$

b) $P(A' \cup B) =$
$= P(A') + P(B) - P(A' \cap B)$
$= 0.5 + 0.6 - 0.4 = \boxed{0.7}$

c) $P(A' \cap B') = P(A \cup B)'$
$= 1 - P(A \cup B) = \boxed{0.1}$

* $P(A') = 1 - P(A)$
$= 1 - 0.5$
$= 0.5$

* $P(B) = P(A' \cap B) + P(A \cap B)$
$= 0.4 + 0.2$
$= 0.6$

* $P(B' \cap A) = P(A) - P(A \cap B)$
$= 0.5 - 0.2$
$= 0.3$

d) $P(B' / A) =$
$$\frac{P(B' \cap A)}{P(A)} = \frac{0.3}{0.5} = \boxed{0.6}$$

Are the events B and A' independent? Explain.

$P(A' \cap B) \stackrel{?}{=} P(A') \times P(B)$
$0.4 \stackrel{?}{=} 0.5 \times 0.6$
$0.4 \neq 0.3 \Rightarrow$ B and A' are dependent

Are the events B and $A \cap B'$ mutually exclusive? Explain.

$B \cap (A \cap B') = \emptyset \Rightarrow$ B and $A \cap B'$ are mutually excl.

2. Nadia and Kate applied to two different jobs. The probability that Nadia will be hired is 0.65 while the probability that Kate will be hired is 0.58. (9 %)

Assume independence and find the probability that:

a) neither will be hired
$P(N' \cap K') = P(N') \times P(K') = (1 - 0.65)(1 - 0.58)$
$= (0.35)(0.42) = \boxed{0.147}$

b) only one of them will be hired.
$P(N \cap K') + P(N' \cap K) = (0.65)(0.42) + (0.58)(0.35)$
$= 0.273 + 0.203 = \boxed{0.476}$

c) At least one of them will be hired

* $P(N \cup K) = P(N) + P(K) - P(N \cap K)$
$= 0.65 + 0.58 - (0.65)(0.58) = \boxed{0.853}$

OR * b) + $P(N \cap K) = 0.476 + 0.377 = \boxed{0.853}$

3. The probability that a tree planted by a landscaping firm will survive is 0.9.

(20 %)

- If 20 trees are planted find the probability that : **(Don't calculate the final answer)**

a) exactly 15 will survive

$$P(X = 15S) = {}_{20}C_{15} \times 0.9^{15} \times 0.1^{5}$$

b) exactly 3 won't survive

$$P(X = 3S') = {}_{20}C_{3} \times 0.1^{3} \times 0.9^{17}$$

c) at least 2 will survive

$$P(X \geq 2S) = 1 - P(X = 0S) - P(X = 1S)$$
$$= 1 - {}_{20}C_{0} \times 0.9^{0} \times 0.1^{20} - {}_{20}C_{1} \times 0.9^{1} \times 0.1^{19}$$

d) less than 3 won't survive

$$P(X < 3S') = P(X = 0S') + P(X = 1S') + P(X = 2S')$$
$$= {}_{20}C_{0} \times 0.1^{0} \times 0.9^{20} + {}_{20}C_{1} \times 0.1^{1} \times 0.9^{19} + {}_{20}C_{2} \times 0.1^{2} \times 0.9^{18}$$

e) at most 5 will survive knowing that at least 3 survived

$$P(X \leq 5S / X \geq 3S) = \frac{P(3S \leq X \leq 5S)}{P(X \geq 3S)} = \frac{P(X = 3S) + P(X = 4S) + P(X = 5S)}{1 - P(X = 0S) - P(X = 1S) - P(X = 2S)}$$
$$= \frac{{}_{20}C_{3} \times 0.9^{3} \times 0.1^{17} + {}_{20}C_{4} \times 0.9^{4} \times 0.1^{16} + {}_{20}C_{5} \times 0.9^{5} \times 0.1^{15}}{1 - {}_{20}C_{0} \times 0.9^{0} \times 0.1^{20} - {}_{20}C_{1} \times 0.9^{1} \times 0.1^{19} - {}_{20}C_{2} \times 0.9^{2} \times 0.1^{18}}$$

- If 30 trees are planted on the average what is the expected number of trees that will not survive?

$$n = 30 \qquad p(S') = 0.1$$

$$\mu = np = 30 \times 0.1 = 3$$

4. Suppose all fourth graders in a certain school are taught to read by the same teaching method and
23 %) that at the end of the year they are tested for reading speed. Suppose reading speed is normally distributed with mean of 150 words per minute and standard deviation of 25 words per minute.

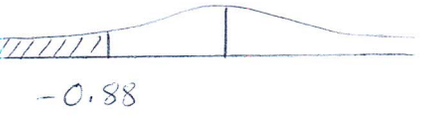
- What is the probability that a randomly selected student reads:

a) Less than 172 words per minute?

$$P(X<172) = P\left(z < \frac{172-150}{25}\right)$$
$$= P(z<0.88)$$
$$= 0.5 + A(0.88)$$
$$= 0.5 + 0.3106$$
$$= 0.8106$$

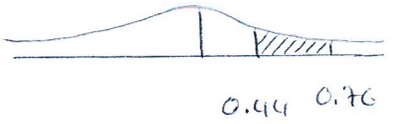
b) At most 128 words per minute?

$$P(X \leq 128) = P\left(z < \frac{128-150}{25}\right)$$
$$= P(z<-0.88)$$
$$= 0.5 - A(0.88)$$
$$= 0.5 - 0.3106$$
$$= 0.1894$$



c) Between 161 and 169 words per minute?

$$P(161<X<169) = P\left(\frac{161-150}{25} < z < \frac{169-150}{25}\right)$$
$$= P(0.44<z<0.76)$$
$$= A(0.76) - A(0.44)$$
$$= 0.2764 - 0.1700$$
$$= 0.1064$$



Continue nb. 4.

- If it is known that the student reads less than 172 words per minute, determine the probability that he reads more than 140 words per minute.

$$P(X>140 / X<172) = \frac{P(140 < X < 172)}{P(X < 172)}$$

$$= \frac{P\left(\frac{140-150}{25} < z < \frac{172-150}{25}\right)}{P\left(z < \frac{172-150}{25}\right)}$$

$$= \frac{P(-0.4 < z < 0.88)}{P(z < 0.88)}$$

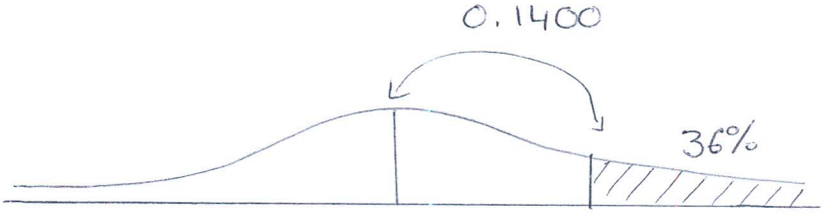
$$= \frac{A(0.4) + A(0.88)}{0.5 + A(0.88)} = \frac{0.1554 + 0.3106}{0.5 + 0.3106} = \frac{0.466}{0.8106}$$

$$= 0.574$$

- If 36 % of the students read more words per minute than Sami does, how many words can Sami read per minute?

$50 - 36 = 14\%$

$0.1400 \approx 0.1406$



$Z_0 = 0.36$

$X_0 = 150 + 0.36 \times 25$

$= 159$

5. In a class of 100 students 60 are business major, 30 are architecture major, and the remaining are chemistry major. 30% of the business students live in the dorm, 20 % of the architecture students live in the dorm while only 10% of the chemistry student don't live in the dorm.

(18 %)

- If one student is selected randomly , what is the probability that the student selected:

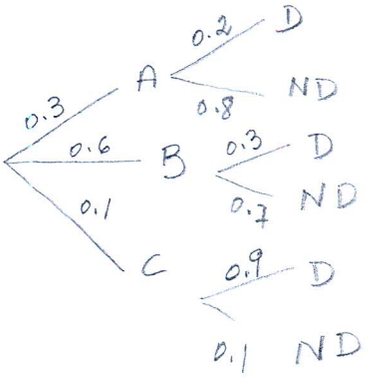
a) is a chemistry student living in the dorm?

$$P(C \cap D) = 0.1 \times 0.9 = \frac{9}{100}$$

	D	ND	Tot
A	6	24	30
B	18	42	60
C	9	1	10
Tot	33	67	100

b) is an architecture student not living in the dorm?

$$P(A \cap ND) = 0.3 \times 0.8 = 0.24 = \frac{24}{100}$$



c) lives in the dorm?

$$P(D) = 0.3 \times 0.2 + 0.6 \times 0.3 + 0.1 \times 0.9 = 0.33$$

$$= \frac{33}{100}$$

d) Is a chemistry student given that he is living in the dorm?

$$P(C \mid D) = \frac{P(C \cap D)}{P(D)} = \frac{0.1 \times 0.9}{0.33} = \frac{9}{33}$$

e) Living in the dorm knowing that he is not a business student

$$P(D \mid B') = \frac{P(D \cap B')}{P(B')} = \frac{P(D \cap A) + P(D \cap C)}{1 - P(B)}$$

$$= \frac{0.3 \times 0.2 + 0.1 \times 0.9}{1 - 0.6} = \frac{6 + 9}{40} = \frac{15}{40}$$

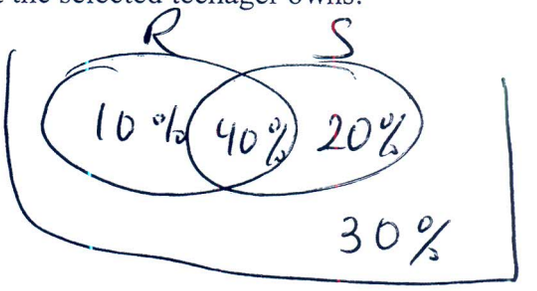
- If 8 students are selected randomly with replacement, what is the probability that exactly 5 live in the dorm?

$$n = 8 \qquad p = P(D) = 0.33 \qquad q = 0.67$$

$$P(X = 5) = {}_8C_5 \, (0.33)^5 \, (0.67)^3$$

6. In New York State, 60 % of all teenagers own skateboards, 40 % own both skateboards and roller blades, 30 % neither own skateboards nor roller blades.

(12 %) If one teenager is selected randomly what is the probability that the selected teenager owns:



a) only a roller blade?

$P(R \cap S') = 0.1$

b) a roller blade given that the teenager owns a skateboard?

$$P(R/S) = \frac{P(R \cap S)}{P(S)} = \frac{0.4}{0.6} = \frac{2}{3}$$

c) a skateboard knowing that he doesn't own a roller blade?

$$P(S/R') = \frac{P(S \cap R')}{P(R')} = \frac{0.2}{0.5} = \frac{2}{5}$$

d) a skateboard or doesn't own a roller blade ?

$$P(S \cup R') = P(S) + P(R') - P(S \cap R')$$
$$= 0.6 + 0.5 - 0.2$$
$$= 0.9$$

7. A green and a red dice are rolled , the product of the numbers that turn up are observed, what is

6 %) the probability that the product is:

a) 12 3,4 ; 4,3 ; 6,2 ; 2,6

$\frac{4}{36}$

b) greater than 25 5,6 ; 6,5 ; 6,6

$\frac{3}{36}$